Reg.No. ____________

**Karunya UNIVERSITY**

(Karunya Institute of Technology & Sciences)

(Declared as Deemed-to-be University under Sec.3 of the UGC Act, 1956)

# End Semester Examination – Nov/Dec – 2016

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| | | | **Semester** | **:** | **2016-17 ODD** |
| **Code** | **:** | **15MA3012** | **Duration** | **:** | **3hrs** |
| **Sub. Name** | **:** | **Functional Analysis** | **Max. marks** | **:** | **100** |

**ANSWER ALL QUESTIONS (5 x 20 = 100 Marks)**

| Q. No. | Sub Div. | Questions | Course Outcome | Marks |
|---|---|---|---|---|
| 1. | a. | Let T be a linear operator. Then prove that<br>a) The range R(T) is a vector space.<br>b) If $\dim D(T) = n < \infty$, then $\dim R(T) \le n$.<br>c) The null space N(T) is a vector space. | CO1 | **10** |
| | b. | Let X, Y be vector space, both real or both complex. Let $T : D(T) \to Y$ be a linear operator with domain $D(T) \subset X$ and range $R(T) \subset Y$. Then prove that<br>a) The inverse $T^{-1} : R(T) \to D(T)$ exists if and only if Tx = 0 $\Rightarrow$x = 0.<br>b) If $T^{-1}$ exists, it is a linear operator. | CO1 | **10** |
| | | **(OR)** | | |
| 2. | a. | If $T : D(T) \to Y$ be a bounded linear operator, where *D(T)* lies in a normed space X and Y is Banach space. Then prove that T has an extension $\tilde{T} : \overline{D(T)} \to Y$ where $\tilde{T}$ is bounded linear operator of norm $\|\tilde{T}\| = \|T\|$. | CO1 | **15** |
| | b. | Prove that finite dimensional vector space is algebraically reflexive. | CO1 | **5** |
| 3. | a. | Prove dual space of $l^p$ is $l^q$, here $1 < p < \infty$ and $\frac{1}{p} + \frac{1}{q} = 1$. | CO2 | **15** |
| | b. | Let X be normed space and let $x_0 \neq 0$ be any element of X. Then prove there exists a bounded linear functional $\bar{f}$ on X such that $\|\bar{f}\| = 1, \bar{f}(x_0) = \|x_0\|$. | CO2 | **5** |
| | | **(OR)** | | |
| 4. | a. | State and prove Baire's Category theorem. And hence prove Uniform Boundedness Theorem. | CO2 | **20** |
| 5. | a. | State and prove Open Mapping Theorem. | CO2 | **20** |
| | | **(OR)** | | |
| 6. | a. | State and prove Banach fixed point theorem. | CO2 | **10** |
| | b. | Let *f(x, y)* be a continuous function of 2 variables in a rectangle $A = \{(x, y) / a \le x \le b, c \le y \le d\}$ and satisfy the Lipschitz condition of order 1 in the second variable y. Further, let $(x_0, y_0)$ be an interior point of A. Then prove that the differential equation $\frac{dy}{dx} = f(x, y)$ has a unique solution say y = g(x) which passes through $(x_0, y_0)$. | CO2 | **10** |
| 7. | a. | A Banach space is a Hilbert Space if and only if its norm satisfies the parallelogram law. | CO3 | **20** |
| | | **(OR)** | | |

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|---|---|---|---|---|
| 8. | a. | State and prove Gram-Schmit Orthogonalization. | CO3 | **10** |
| | b. | Prove that $l_2^n$ is the inner product space but not Hilbert Space for $p \neq 2$ . | CO3 | **10** |
| | | **Compulsory:** | | |
| 9. | a. | State and prove Riesz Representation Theorem. | CO3 | **10** |
| | b. | Define adjoint operator. Prove that adjoint operator always exists, bounded, linear and unique. | CO3 | **10** |

ALL THE BEST